

**CONTROL SYSTEM ENGINEERING - II**

*Full Marks : 70*

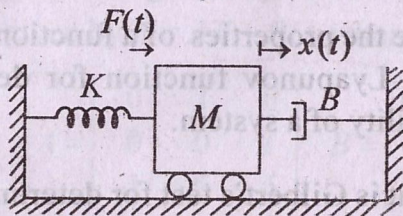
*Time : 3 hours*

**Answer Q. No. 1, which is compulsory and any five from the rest**

*The figures in the right-hand margin indicate marks*

**1. Answer the following : 2 × 10**

- (a) What do you mean by a modal matrix ? Given any system matrix, how can you construct a modal matrix ?
- (b) Obtain the state model of the mechanical system shown in figure.



( Turn Over )

( 2 )

- (c) Obtain the phase variable state model for the system described by the differential equation

$$4\frac{d^3y}{dt^3} + 3\frac{d^2y}{dt^2} + \frac{dy}{dt} + 2y(t) = 5u(t)$$

- (d) What is meant by 'Pole placement through state feedback control'? How does it differ from the design approach by root locus method?

- (e) What do you mean by a 1st order hold device? How does it differ from a zero order hold?

- (f) Show that the mapping  $z = e^{ST}$  between the S-plane and the Z-plane is not a one-to-one mapping.

- (g) State the properties of a function to qualify as a Lyapunov function for determining stability of a system.

- (h) What is Gilbert's test for determining state controllability?

( 3 )

- (i) Write any three differences between a lag and a lead compensator.

- (j) What do you understand by the terms 'phase plane' and 'phase trajectory'?

2. The state equation of a linear time-invariant system is given as

$$\dot{X} = \begin{bmatrix} 0 & 5 \\ -1 & -2 \end{bmatrix} X + \begin{bmatrix} 1 \\ 1 \end{bmatrix} r \quad \text{and} \quad Y = \begin{bmatrix} 1 & 1 \end{bmatrix} X$$

Determine

4 + 4 + 2

- (a) The characteristic equation and eigenvalues  
(b) The state transition matrix  
(c) The transfer function

Also, draw the state diagram.

3. (a) Consider a regulator system in which the plant is given by  $\dot{X} = AX + BU$ , where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

( 4 )

The system uses state feedback control  $u = -kx$  to locate the desired closed loop poles at

$$S_1 = -2 + j4, S_2 = -2 - j4, S_3 = -10$$

Determine the state feedback gain matrix  $K$  using any one method.

5

(b) Draw the block diagram of an observed state feedback control system and show that the controller poles and the observer poles can be placed independent of each other.

5

4. (a) Obtain the inverse z-transform of

$$X(z) = \frac{z^2}{(z-1)^2(z-e^{-at})}$$

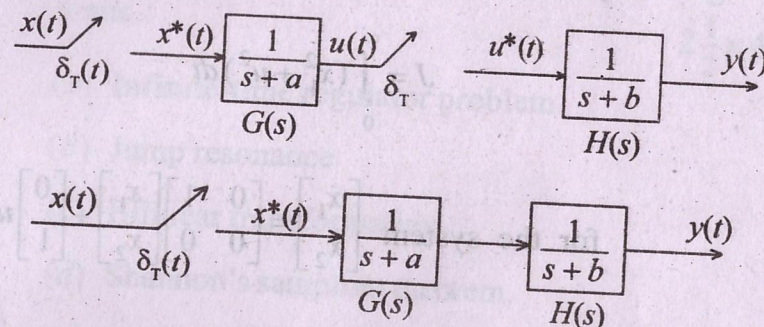
by using (i) inversion integral method

(ii) partial fraction expansion method.

6

( 5 )

(b) Consider the systems shown below



Show that the two systems have different pulse transfer functions.

4

5. (a) Obtain the Z-transform of  $K^3$ .

3

(b) Derive the formula to determine the Z-transform of a sequence  $X(k)$ , advanced by  $n$  sampling instants.

3

(c) A continuous time system has state equation given by

$$\dot{X} = AX, \text{ where } A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$$

Find  $F(A) = e^{AT}$  using Cayley-Hamilton theorem.

4

( 6 )

6. (a) Obtain the control law which minimizes the performance index

$$J = \int_0^{\infty} (x_1^2 + u^2) dt$$

for the system  $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$  5

- (b) Explain in brief about the various non-linearities. Draw the input-output characteristic in each case. 5

7. (a) What do you understand by the term 'describing function'? Determine the describing function for the saturation non-linearity. 6

- (b) What is a lead compensator? Show its pole-zero location in the  $s$ -plane. Derive an expression for the maximum phase lead and the frequency at maximum phase lead. 4

( 7 )

8. Give brief description about the following terms :  $2\frac{1}{2} \times 4$

- (a) Infinite-time regulator problem
- (b) Jump resonance
- (c) Bilinear transformation
- (d) Shannon's sampling theorem.